SYLLABUS: 1 JANUARY 1994

REPUBLIC OF SOUTH AFRICA

CO-ORDINATOR: ENGINEERINGSTUDY

SYLLABUS FOR

MATHEMATICS N3

NATIONAL CERTIFICATE

CODE NUMBER

9213

EXAMINATION INSTRUCTION NO. 4 OF 1994

DATE OF IMPLEMENTATION
AUGUST 1994

DATE OF FIRST EXAMINATION
NOVEMBER 1994

1. SUBJECT AIMS AND OBJECTIVES FOR MATHEMATICS

1.1 AIMS

Students should understand the mathematical principles taught in each module in such a way that they will be able to apply these principles in Engineering Science or the trade theories which they study.

1.2 OBJECTIVES

To teach the student in such a manner that, on completion of all the modules in Mathematics N3, he will be able to

* apply the mathematical principles mastered by him to his specific trade theory;
* use the correct mathematical terminology and to identify the appropriate formulae;
* use the correct SI units and derived units;
* commence with the study of Mathematics N4;
* apply the basic mathematical principles he has mastered in the working-place and in everyday life;
* reason logically when seeking solutions to mathematical and scientific problems; and
* function effectively in his working environment and to make sense of the extended technology that he is confronted with.

2. DURATION OF INSTRUCTIONAL OFFERING

The duration of this instructional offering is one trimester full-time (10 weeks/75 hours) or one trimester part-time (10 weeks/60 hours), which includes revision, tests and examinations.

3. EVALUATION

Candidates should be evaluated on a regular basis by conducting class tests on completion of each module.

4. EXAMINATION

4.1 Reproducing, application, analysing and evaluation are important aspects in order to determine the levels of difficulty in this subject, and the division thereof should be as follows:

REPRODUCING	APPLICATION	ANALYSING	EVALUATION
40	25	20	15

4.2 A paper of 3 hours to the value of 100 marks is set at the end of each trimester.

4.3 Only content classified as **LEARNING OUTCOMES** will be examined.

5. GENERAL INFORMATION

5.1 In order to bring the student, where possible, into contact with the situation in practice, problems must be taken from the practical situation.

5.2 The correct use of the suitable technical terminology must be stressed, especially in formulating definitions and principles.

5.3 Calculation answers must be approximated to three decimals. Approximation may not take place during calculations - only the final answer must be approximated.

5.4 When a standard formula is used in a calculation, the formula must be written down first before the values in the formula are substituted. If any manipulation is applied it must be clearly indicated.

5.5 Pocket calculators may be used in solving mathematical problems. Basic instruction must be offered in the practical use and operational abilities of the calculator.

5.6 The weight value (WV) indicates the time which should be spent to conclude a module as well as the approximate weight the module will carry in the examination.

5.7 Exposition of subject matter

The theme is preceded by the word MODULE, followed by a number indicating its chronological position. Decimal numbers indicate the CONTENT to be dealt with, and extended decimal numbers identify the expected LEARNING OUTCOMES.

6. SUBJECT MATTER

MODULE		WEIGHT VALUE
6.1	Factors and fractions	(19)
6.2	Exponents, surds and logarithms	(13)
6.3	Equations, word problems and manipulations	(14)
6.4	Geometry of co-ordinates	(18)
6.5	Algebraic graphs	(9)
6.6	Differential calculus	(7)
6.7	Trigonometry	(20)
		(100)

7. DETAILED SYLLABUS

MODULE 1: FACTORS AND FRACTIONS

1.1 THE COMMON FACTOR

DIDACTIC GUIDELINE

Before the presentation of this topic, grouping must first be revised.

On completion of this topic, the student should be able to:

1.1.1 Factorise polynomials by looking for the common factor.

1.2 THE QUADRATIC TRINOMIAL

On completion of this topic, the student should be able to:

1.2.1 Factorise a quadratic trinomial. There are no limitations on any of the terms. A common factor may also be included, e.g. $3a^2 + 6ab + 3b^2$.

1.3 THE DIFFERENCE OF TWO SQUARES

On completion of this topic, the student should be able to:

1.3.1 Factorise the difference of two squares. The two squares may be not more than binomials, while a common factor may also be included, e.g.

(i) $3a^2 - 27b^2$
$= 3(a^2 - 9b^2)$
$= 3(a - 3b)(a + 3b)$

(ii) $25 - (a + b)^2$
$= [5 - (a + b)][5 + (a + b)]$

1.3.2 Solve quadratic equations by completing the square

1.3.3 Derive the quadratic formula by completing the square.

1.4 CUBE FUNCTIONS

On completion of this topic, the student should be able to:

1.4.1 Apply the residue and factor theorems to cube functions.

1.5 ALGEBRAIC FRACTIONS

On completion of this topic, the student should be able to:

1.5.1 Do multiplication and division of algebraic fractions by first factorising the numerators and denominators

1.5.2 Add and subtract algebraic fractions (the limitations on numerators and denominators are the same as for factorisation. Changing of the signs after factorisation is included to make it easier to determine the LCF.)

1.5.3 Simplify fractions over fractions where there are only a maximum of two fractions in the numerator and/or the

denominator (the fractions may contain constants or variables), e.g. $\frac{a/b + c/d}{d/c - c/b}$.

MODULE 2: EXPONENTS, SURDS AND LOGARITHMS

2.1 EXPONENTS

On completion of this topic, the student should be able to:

2.1.1 Apply the laws of exponents and their derivatives to solve algebraic expressions (the exponents may be positive or negative integers or positive or negative fractions.)

2.1.2 Solve equations of the following forms in which the radices are only positive real numbers, e.g.
(i) $3x^{1/3} = 27$
(ii) $3^{x^2 + 5x + 6} = 1$
(iii) $2^x + 3.2^x = 12$ - Common factor.

DIDACTIC GUIDELINE

Equations of the type $3^{2x} + 5.3^x - 14 = 0$ which must be solved by factorisation (quadratic trinomial) are excluded.

2.2 SURDS

On completion of this topic, the student should be able to:

2.2.1 Simplify expressions with surds by using the rules of the four basic operations, factorisation and involution

2.2.2 Rationalise fractions with irrational numerators (surds in the numerators) where the irrational numerators may only be monomials

2.2.3 Solve equations containing surds where only equations in which it is necessary to square once are allowed, e.g. $\sqrt{(x + 5)} = x + 2$

2.2.4 Solve equations which can be converted to quadratic equations, e.g.

$$\frac{1}{2x - 1} + \frac{x}{2x + 1} = \frac{2}{4x^2 - 1}.$$

DIDACTIC GUIDELINE

All radices are limited to positive real numbers.

2.3 LOGARITHMS

On completion of this topic, the student should be able to:

2.3.1 Reproduce the four laws of logarithms, viz.
(i) $\log_a xy = \log_a x + \log_a y$
(ii) $\log_a x/y = \log_a x - \log_a y$
(iii) $\log_a x^m = m.\log_a x$
(iv) $\log_a b = \log_c b/\log_c a$

2.3.2 Apply the laws of logarithms to simplify expressions

2.3.3 Solve logarithmic equations in which the natural logarithm $\log_e x$ or ln x appears, with specific reference to the change in the radix.

MODULE 3: EQUATIONS, WORD PROBLEMS AND MANIPULATIONS OF TECHNICAL FORMULAE

3.1 EQUATIONS

On completion of this topic, the student should be able to:

3.1.1 Solve linear equations containing fractions
3.1.2 Solve simultaneous linear equations containing fractions
3.1.3 Solve quadratic equations by means of the quadratic formula or factorisation, keeping the limitations which apply to factorisation in mind
3.1.4 Solve simultaneous equations of which one is linear and the other one is quadratic, in an algebraic manner.

3.2 WORD PROBLEMS

On completion of this topic, the student should be able to:

3.2.1 Set and solve quadratic and/or linear equations from word problems - the equations may not both be quadratic.

3.3 MANIPULATION OF TECHNICAL FORMULAE (CHANGING THE SUBJECT OF THE FORMULAE)

On completion of this topic, the student should be able to:

3.3.1 Change the subject of a given formula to any other required subject. The following applications are *included*:

(i) Manipulation of exponents, e.g.
$T = ar^{n-1}$ (n)

(ii) Manipulation by factorisation, e.g.
$E = \frac{1}{2}mv^2 - \frac{1}{2}mu^2$ (m)

(iii) Manipulation by using the quadratic formula, e.g.
$s = ut + \frac{1}{2}at^2$ (t)

(iv) Manipulation of cubes, squares, square roots and cube roots

3.3.2 Determine the value from advanced technical formulae by manipulation and substitution.

MODULE 4: GEOMETRY OF CO-ORDINATES

4.1 THE STRAIGHT LINE

On completion of this topic, the student should be able to:

4.1.1 Find the equation of a straight line if one of the following sets of information is given:
- (i) A point on the line and the gradient of the line
- (ii) Two points on the line
- (iii) The equation of a line parallel to the required line and a point on the line
- (iv) The equation of a line perpendicular to the required line and a point on the line

4.1.2 Write the calculated equation of the line in the following manners:
- (i) Common form: $ay + bx + c = 0$
- (ii) Gradient offcut form: $y = mx + c$
- (iii) Intercept form: $x/a + y/b = 1$
- (iv) Gradient point form: $m = (y - y_1)/(x - x_1)$
- (v) Two-point form: $(y-y_1)/(x-x_1) = (y_2-y_1)/(x_2-x_1)$

4.1.3 Determine the length of a line segment between two given points

4.1.4 Determine the angle of inclination of a straight line

4.1.5 Determine the gradient of a straight line from a given angle of inclination

4.1.6 Determine the co-ordinates of the midpoint of a given line segment

4.1.7 Determine the equation of a circle with its centre at the origin and given radius

4.1.8 Determine the points of intersection of a given circle (centre at origin) and a straight line

4.1.9 Determine the equation of a tangent in a given direction to the circle

4.1.10 Determine the equation of a tangent to the circle at a specified point on the circumference of the circle.

MODULE 5: ALGEBRAIC GRAPHS

5.1 SKETCH GRAPHS

On completion of this topic, the student should be able to:

5.1.1 Draw the following graphs if the equations are given:
- (i) The straight line: $y = mx + c$
- (ii) The circle: $y = \pm\sqrt{(r^2 - x^2)}$ [r^2 a complete square]
- (iii) Semi-circles
- (iv) The rectangular hyperbola: $xy = a$
- (v) The parabola: $y = ax^2 + bx + c$ and $x = ay^2$
- (vi) The ellipse: $x^2/a^2 + y^2/b^2 = 1$
- (vii) The cube function $y = ax^3 + bx^2 + cx + d$, where $d = 0$, or if $d \neq 0$, then there should be at least one root ≤ 0 and an integer.

N.B. Where applicable, the intercepts with the axes should be shown.

5.2 GRAPHIC SOLUTIONS

On completion of this topic, the student should be able to:

5.2.1 Graphically solve simultaneous equations of any two graphs mentioned in 5.1.1

5.2.2 Draw the graph of $y = ax^n$ and indicate the influence on the form of the graph with changes in the values of a and n.

MODULE 6: DIFFERENTIAL CALCULUS

6.1 AVERAGE GRADIENT AND AVERAGE SPEED

On completion of this topic, the student should be able to:

6.1.1 Calculate the average gradient of a curve between the points (x_1,y_1) and (x_2,y_2) from $M_{ave} = y_2 - y_1/x_2 - x_1$

6.1.2 Calculate the average speed of an object

6.2 LIMITS

On completion of this topic, the student should be able to:

6.2.1 Determine the $\lim_{h\to 0} \frac{f(x + h) - f(x)}{h}$, where f must be chosen from the following functions: k, ax, ax + b and ax^2

6.2.2 Indicate that the following symbols denoting the derivative of a function, all have the same meaning: D_x, d/dx and f'(x), and if y = f(x), then dy/dx.

6.3 RULES FOR DIFFERENTIATING

On completion of this topic, the student should be able to:

6.3.1 Indicate the following rules for differentiating, and apply them to simple problems:
(i) $D_x[x^n] = nx^{n-1}$, n a real number
(ii) $D_x[f(x) \pm g(x)] = D_x[f(x) \pm D_x[g(x)]$
(iii) $D_x[kf(x)] = kD_x[f(x)]$

6.3.2 Determine the gradient of a curve at any point of the curve

6.3.3 Calculate the turning points of polynomials of at most the third degree and make a sketch of the curve (maxima, minima and points of inflection are excluded). Polynomials of the third degree will contain no constant, thus $y = ax^3 + bx^2 + cx$.

DIDACTIC GUIDELINE

An intuitive approach to the concept of a limit should be adopted when this theme is first introduced to the students. All the rules for differentiating are accepted without proof.

MODULE 7: TRIGONOMETRY

7.1 EQUATIONS AND EXPRESSIONS

On completion of this topic, the student should be able to:

7.1.1 Solve linear trigonometric equations for angles in one revolution (equations with a common factor are included.)

7.1.2 Simplify trigonometric expressions by using exact values

7.1.3 Solve acute-angled and obtuse-angled triangles which result from the application of inaccessible heights and distances by making use of the sine, cosine and the area rules

7.1.4 Simplify trigonometric expressions by applying inverse, quotient and square identities.

7.2 GRAPHS

On completion of this topic, the student should be able to:

7.2.1 Draw sketch graphs of the following trigonometric functions by making use of the amplitude period method:

(i) $y = a.\sin x$
(ii) $y = a.\cos x$
(iii) $y = \sin bx$
(iv) $y = \cos bx$
(v) $y = a.\sin bx$
(vi) $y = a.\cos bx$
(vii) $y = a.\sin (bx + c)$
(viii) $y = a.\cos (bx + c)$

7.2.2 Read approximate values from the graph after it has been drafted - limited to not more than two graphs on the same axial system

7.2.3 Develop sinusoidal waveforms from a vector rotating with an angular velocity of ω radians per second

7.2.4 Combine two trigonometric waves (functions) on the same axial system.