AQP for Foundational Learning Competence

FOUNDATIONAL LEARNING COMPETENCE MATHEMATICAL LITERACY: CURRICULUM FRAMEWORK

CONTENTS	
A. INTRODUCTION TO THE FOUNDATIONAL LEARNING COMPETENCE MATHEMATICAL LITERACY FRAMEWORK	Sets out the definition, overall purpose, elements, levels of complexity and scope of contexts of Foundational Mathematical Literacy
B. THE FOUNDATIONAL LEARNING COMPETENCE MATHEMATICAL LITERACY FRAMEWORK	Sets out the organising principles, curriculum elements and outcomes, including descriptions of scope and contexts
C. FACILITATOR GUIDELINES, ASSESSMENT GUIDELINES AND ACCREDITATION SPECIFICATIONS	Gives general guidelines on methodology and assessment

SECTION A
INTRODUCTION TO THE FOUNDATIONAL LEARNING COMPETENCE MATHEMATICAL LITERACY FRAMEWORK

1. DEFINITION OF FOUNDATIONAL MATHEMATICAL LITERACY

Foundational Mathematical Literacy (FML) is the minimum, generic mathematical literacy that will provide learners with the necessary foundation to:

- access learning at National Qualifications Framework (NQF) Levels 2, 3 and 4 for occupations and trades; and
- engage meaningfully in real-life situations.

Important note: Where particular occupations or trades require mathematics beyond FML, or applications in contexts that are specific to occupations or trades, these requirements are addressed within the core of qualifications related to those occupations and trades.

2. OVERALL PURPOSE OF FML

People who have met all the requirements of FML are able to:

Make sense of and solve problems in real contexts by responding to information about mathematical ideas that is presented in a variety of ways.

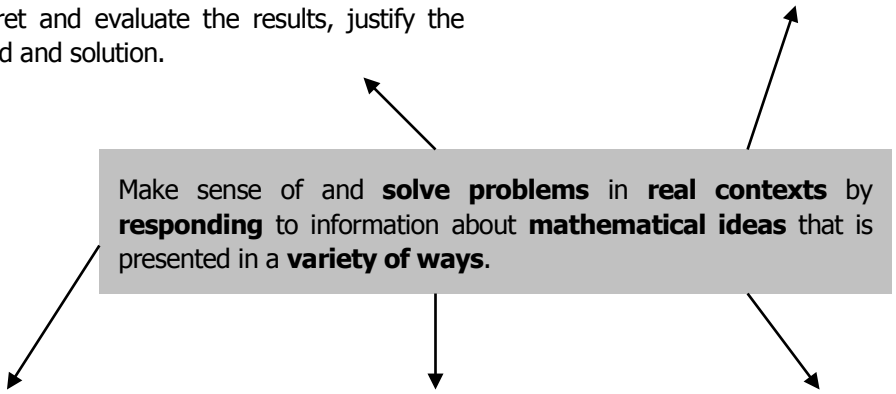
This overall purpose of FML is clarified below:

Solving problems means that the learner can:

- Define the goals to reach.
- Analyse and make sense of the problem situation.
- Plan how to solve the problem.
- Execute the plan.
- Interpret and evaluate the results, justify the method and solution.

Real contexts means:

- In everyday life.
- At work.
- In further learning.



Responding means:

- Identifying or locating.
- Acting upon.
- Ordering, sorting and comparing.
- Counting.
- Estimating.
- Computing.
- Measuring.
- Modelling.
- Interpreting.
- Communicating about.

Variety of ways means in:

- Objects and pictures.
- Numbers and symbols.
- Formulae.
- Diagrams and maps.
- Graphs.
- Tables and spreadsheets.
- Texts.

Mathematical ideas means:

- Number and quantity.
- Space and shape.
- Patterns and relationships.
- Data and chance.
- Measurements.

3. ELEMENTS OF FML AND THEIR PURPOSES

Element	Purpose
1. Number	Use numbers in a variety of forms to describe and make sense of situations, and to solve problems in a range of familiar and unfamiliar contexts.
2. Finance	Manage personal finances using financial documents and related formulae
3. Data and chance	Collect, display and interpret data in various ways and solve related problems.
4. Measurement	Make measurements using appropriate measuring tools and techniques and solve problems in various measurement contexts.
5. Space and shape	Describe and represent objects and the environment in terms of spatial properties and relationships.
6. Patterns and relationships	Describe, show, interpret and solve problems involving mathematical patterns, relationships and functions.

4. LEVELS OF FML

In keeping with the generally accepted and broad definition of mathematical literacy as the application of basic mathematics in different real-life contexts, the following statements about levels can be made about FML:

- The level of mathematics involved is roughly equivalent to NQF Level 1.
- The level of complexity of the contexts ranges from NQF Level 1 to NQF 4. Learners need to straddle these levels as demanded by particular situations defined in the FML Framework.

Although the framework gives examples of the sort of contexts in which learners are required to apply mathematics, it makes no distinctions between the levels of those contexts. Therefore, the level of FML is not pegged at any particular level but is regarded as foundational to learning for occupations and trades at NQF Levels 1 – 4.
From an administrative point of view, the FML Curriculum is part of the Foundational Learning Competence Part Qualification at NQF Level 2.

5. SCOPE OF CONTEXTS

The FML Framework identifies a range of contexts as a guide to the sort of contexts that could be used in learning and assessment activities. This does not mean that the list is complete, and it is possible that the range of contexts will be expanded in later drafts.

The framework does not try to cover the full range of contexts that people will meet in life, work and learning. The framework is based on the assumption that particular occupational contexts will be addressed within those particular occupations.

However, FML programmes should expose learners to a sufficiently wide range of contexts in order to ensure a solid foundation. The programmes should also help learners to develop their skills of transferring what they know from familiar to unfamiliar contexts. Therefore, when learners find themselves in unfamiliar contexts, whether in the world of work or within occupational learning areas, they should be able to respond to further learning within those new contexts with some coaching.

6. LEARNING ASSUMED TO BE IN PLACE

It is assumed that learners starting FML learning programmes are already competent in communications and in mathematical literacy at Adult Basic Education and Training (ABET) Level 3.

SECTION B
THE FML FRAMEWORK

CLARIFICATION OF ELEMENTS: PURPOSE, SKILLS, REQUIRED STANDARDS OF PERFORMANCE, EXAMPLES OF CONTEXTS, GUIDELINES, SCOPE AND CONTEXTS

Element 1: Number

Purpose: Use numbers in a variety of forms to describe and make sense of situations, and to solve problems in a range of familiar and unfamiliar contexts.

Skills:

1.1 Use numbers to makes sense of and describe situations
1.2 Read, interpret and use different numbering conventions in different contexts and identify the ways in which different conventions work
1.3 Interpret and use numbers written in exponential form including squares and cubes of natural numbers and the square and cube roots of perfect squares and cubes
1.4 Do calculations in various situations using a variety of techniques
1.5 Solve problems involving ratio and proportion
1.6 Solve problems involving fractions, decimals and percentages

Required Standards of Performance:

- Problem-solving strategies are based on a correct interpretation of the context.
- Calculations are performed accurately and according to the conventions governing the order of operations.
- Methods are presented in a clear, logical and structured manner, using mathematical symbols and notation consistent with mathematical conventions.
- Methods used are efficient, logical, internally consistent and justified by the context.
- Solutions are evaluated and validated in terms of the context, and numbers rounded appropriately to the problem situation.

SKILLS		GUIDELINES, SCOPE AND CONTEXTS

1.1	Use numbers to makes sense of and describe situations	• Understand number concept and its associated language.
		• Do basic mental calculations.
		• Use numbers to count, order and estimate.
		• Use names and symbols for numbers.
		• Understand subsets of whole numbers (odd, even, square, square roots, factors and multiples).
		• Use positive and negative numbers as directional indicators:
		  o –10 °C to indicate 10 °C below freezing.
		  o –R300 to indicate overdraft.
		  o –40 m to indicate 40 m below sea level.
		• Add and subtract positive and negative numbers within real contexts e.g., temperature, deposits and overdrafts in a bank.
		• Use fractions to estimate size or portion (such as 'we have used ¼ of our allocated time').

1.2	Read, interpret and use different numbering conventions in different contexts and identify the ways in which different conventions work	• Use the decimal comma and point and the thousands separator correctly:
		  o 2,567,890.00.
		  o 2 567 890,00.
		  o 13.1 in cricket refers to 13 overs and 1 ball of the next over. This is not the same as 13,1 in the decimal system.
		• Use numerals and words interchangeably to indicate number: 1 502 000 = one million five hundred and two thousand.
		• Read and interpret numbers communicated in scientific notation for very small and very big numbers e.g., $4{,}56 \times 10^7 = 45\,600\,000$.
		• Compare and use different time notations:
		  o a.m./p.m.
		  o 24-hour clock.
		  o Analogue and digital.

1.3	Interpret and use numbers written in exponential form including squares and cubes of natural numbers and the square and cube roots of perfect squares and cubes	• $3^2 = 9$. • $2^3 = 8$. • $\sqrt{9} = 3$. • $\sqrt[3]{8} = 2$.

SKILLS	GUIDELINES, SCOPE AND CONTEXTS
1.4 Do calculations in various situations using a variety of techniques	• Estimate solutions to calculations and carry out mental calculations by breaking up and recombining numbers: R234,60 + R419,70 = ± R650. • Apply distributive [4(6 + 3) = 4.6 + 4.3] and associative properties [(2.3). 4 = 2.(3.4)] to simplify calculations where possible and/or useful. (It is not necessary to know the names of the properties.) • Work flexibly with numbers to break up and recombine them to simplify calculations. • Do calculations correctly using techniques appropriate to the problem. These could include paper, mental and/or calculator methods, spreadsheets and standard or non-standard algorithms. Spreadsheets could be an option but not an automatic tool. • Apply the correct order of operations in calculations as per conventions i.e., BODMAS and Left-to-Right rule for operations of the same order e.g., $12 \div 4 \times 2 = 3 \times 2 = 6$; $4 - 3 + 5 = 1 + 5 = 6$. • Use the following functions on a basic calculator (a four-function calculator): o addition, subtraction, multiplication and division; o percentage; o square root; o memory; and o 'clear' and 'clear all' keys. • Estimate reasonable values for unknowns in problems based on experience of the context in order to solve the problems and/or predict a solution. • Round numbers up, down or off (i.e., truncated) appropriately according to the context or instruction. Learners must show that they are aware of the implications of rounding too early or too late in a calculation. • Use the laws of exponents correctly. Learners are not expected to manipulate expressions involving these laws, but should be able to apply these laws when they appear in numerical examples: o $a^n \times a^m = a^{n+m}$. o $a^n \div a^m = a^{n-m}$. o $a^0 = 1$. o $a^{-n} = 1/a^n$.

SKILLS	GUIDELINES, SCOPE AND CONTEXTS

1.5 Solve problems involving ratio and proportion	• Share according to a given ratio.
	  ○ e.g., Mary worked for 2 hours, Susan worked for 3 hours. They earn R400 between them. Share the money according to the hours they worked.
	• Mix according to a given ratio.
	  ○ e.g., A chemical is made up of A and B in the ratio of 2:5. How much of A and B are needed to make 3 litres of the chemical?
	• Increase or decrease amounts according to a given ratio.
	  ○ e.g., Increase R450 by one fifth.
	• Determine a ratio between quantities.
	  ○ e.g., there are 60 men and 75 women in a room. What is the ratio of men to women?
	• Convert between units without losing the ratio: R to $, mm to km.
	• Direct proportion.
	  ○ e.g., car rental is charged per day. The more days you hire the car, the more it costs, in direct proportion to the daily rate.
	• Inverse proportion.
	  ○ e.g., the more people we have on a job, the less time it takes.

1.6 Solve problems involving fractions, decimals and percentages	• Express one quantity as a fraction, percentage and/or decimal of another quantity.
	• Express parts of a whole as fractions, percentages and decimals of a whole.
	• Find a given simple fraction, percentage or decimal of a quantity or shape.
	• Change a quantity or shape by increasing or decreasing it by fractions, percentages or decimals e.g., an item costs R400 excluding VAT, how much does it cost including VAT?
	• Find out the original amount or size of a quantity or shape when given the increased or decreased amount or shape and the fraction, percentage or decimal increase or decrease e.g., an item costs R325 including 14% VAT. How much was the item before VAT?
	• Describe situations or changes by expressing and calculating one quantity as a fraction of another, and recognise situations for using these descriptions.
	• Describe situations involving parts of a whole and recognise situations for using these descriptions.
	• Use fractions, decimals and percentages as measures of parts of a whole:
	  ○ ¾ of the mixture is water.
	  ○ 75% of the mixture is water.
	  ○ 0,75 of the mixture is water.
	• Express fractions in equivalent forms and convert between fractions, decimals and percentages with and without a calculator:
	  ○ $\frac{3}{4}=\frac{6}{8}=\frac{75}{100}=75\%=0{,}75.$

Element 2: Finance

Purpose: Manage personal finances using financial documents and related formulae.

Skills:

2.1 Read and interpret financial information presented in a range of documents in personal and familiar contexts
2.2 Identify, classify and record sources of income and expenditure
2.3 Plan and monitor personal finances
2.4 Evaluate options when purchasing products and services
2.5 Determine the impact of interest, depreciation, inflation, deflation and taxation on personal finances

Required Standards of Performance:

- Interpretations of personal financial documents are consistent with recorded facts.
- Financial information is recorded and organised clearly, accurately and according to general finance-recording techniques and principles.
- Personal budgets reflect the financial situation in sufficient detail for planning and monitoring personal finances.
- Costs of products and services are evaluated using a variety of issues. These include affordability, personal needs and accuracy of advertising claims.
- Personal finances are monitored in terms of various influences, including income, expenditure, investments, loans, taxation and inflation.

SKILLS	GUIDELINES, SCOPE AND CONTEXTS
2.1 Read and interpret financial information presented in a range of documents in personal and familiar contexts	• Documents include: o Pay slips (including deductions and increases). o Cheques. o Receipts. o Bank statements. o Accounts or invoices. • Financial information includes: o Balances. o Credit (income) and debit (expense) items. o Beneficiaries or recipients. o Payments or credits owing. o Date or time periods of documents.
2.2 Identify, classify and record sources of income and expenditure	• Sources of income could include: o Salaries, wages and commissions. o Gifts and pocket money. o Bursaries and loans. o Savings. o Interest. o Inheritances. • Expenditure could include: o Living expenses (e.g., food, rental and clothing). o Accounts (e.g., municipal services). o Communication (e.g., telephone and email). o Fees (e.g., college and bank fees). o Insurance (e.g., medical aid, personal liability and vehicle insurance). o Personal taxes. o Loan repayments (e.g., bank loans, bursaries and hire purchase, including interest). o Entertainment.
2.3 Plan and monitor personal finances	• Income and expenditure statements. • Personal budgets (general and for personal projects, trips, events, significant purchases and others). • Costing of items (such as roofing tiles for a particular area, appliances or painting costs).

SKILLS	GUIDELINES, SCOPE AND CONTEXTS
2.4 Evaluate options when purchasing products and services	• Compare credit cards with different interest rates for different periods of time. • Estimate the long-term costs of making lower monthly credit card payments. • Evaluate advertising claims for products and services and make decisions on best purchase options e.g., compare cell phone packages. • Evaluate different saving and purchasing options. These will include saving accounts, loans, hire purchase, buying on credit and mortgage bonds.
2.5 Determine the impact of interest, depreciation, inflation, deflation and taxation on personal finances	• Compare simple and compound interest. • Explore the effect of depreciation. • Investigate the variables involved in loans and investments e.g., interest rates, duration. • Identify the impact of inflation and deflation on personal finances by comparing the buying power of income from year to year. • Identify tax liability associated with different levels of income by using tax tables. For assessment purposes, learners should be given the relevant formulae. During learning activities, the use of formulae should not be at the expense of developing their understanding through investigating the impacts by using pen and paper and/or spreadsheets and other tools.

Element 3: Data and Chance

Purpose: Collect, display and interpret data in various ways and solve related problems.

Skills:

3.1 Collect data
3.2 Classify, organise and summarise data
3.3 Display data
3.4 Analyse and interpret data to draw conclusions and make predictions
3.5 Determine and interpret chance

Required Standards of Performance:

- Data collection techniques are appropriate to the context, type of data and purpose.
- Data is classified, organised and summarised appropriately so that it promotes meaningful analysis.
- Data is displayed using techniques appropriate to the type of information and context.
- Data displays are consistent with collected data, promote ease of interpretation and minimise bias.
- Interpretations and predictions are verified by the data and observed trends. They take into account possible sources of error and data manipulation.
- Simple probabilities are determined accurately and statements of chance are correctly interpreted in context.

Examples of contexts:

- Human rights, social, economic, cultural, environmental and political matters are examples. They can be used to:
  - carry out quality control by using a representative sample on which to base decisions.
  - write a report about a quantitative issue and include tables and charts. Make a presentation on the material to peers or superiors.
  - read and listen to reports critically and ask intelligent questions.
  - produce a schedule or tree diagram for a project.
  - critically interpret statistical charts and graphs presented in the media.
  - use spreadsheets to investigate different sales options and prepare graphs that illustrate these options.
  - look for patterns in data to identify trends in costs, sales and demand.
  - read statistical tables in reports of local sporting events and understand how they were generated.
  - apply knowledge of probability to risks in health and financial (such as gambling) issues affecting the local community.
  - play games of chance, understanding scoring and statistics.
  - judge the likelihood of an event or story, like that of an Unidentified Flying Object (UFO), being true.

SKILLS	GUIDELINES, SCOPE AND CONTEXTS
3.1 Collect data	• Identify sources of data (e.g., peers, family, newspapers, books, magazines or internet). • Use appropriate data collection techniques (observing, interviewing, administering written questionnaires, conduct focus group discussions, conduct surveys, carry out experiments) and instruments or instruments (e.g., checklist; data forms; measuring instruments; interview guide; checklist; questionnaire; tape recorder; questionnaire). • Use accepted sampling techniques. • Understand sources of bias in data collection. • Understand ethical considerations.
3.2 Classify, organise and summarise data	• Classify collected information as: o Categorical (e.g., male/female; dog/cat; type of car). o Quantitative (numerical): – Discrete (e.g., number of siblings or number of pairs of shoes) – Continuous (e.g., height, weight and rainfall). o Qualitative (not numerical). • Organise data using tally tables, frequency tables, two-way tables or stem-and-leaf diagrams. • Analyse ungrouped data using mode, median and mean. Learners should be able to interpret these measures (e.g., the median is the mid-point of a distribution. Half the data is below the median and half is above it). • Organise data into appropriate class intervals where necessary. • Calculate relative frequency and percentage frequency.

SKILLS	GUIDELINES, SCOPE AND CONTEXTS
3.3 Display data	• Pie charts. • Bar graphs (single bar and multiple bar graphs). • Line and broken-line graphs.
3.4 Analyse and interpret data to draw conclusions and make predictions	• Read and critically interpret data in tally tables, frequency tables, two-way tables, stem-and-leaf diagrams, bar graphs, double bar graphs, pie charts and line graphs. • Choose the most suitable measure of central tendency (mean, median or mode). • Make use of measures of spread (range, quartiles, inter-quartile range). • Show an awareness of the sources of error and data manipulation (grouping, scale and choice of summary statistics). • Explain the misuse of scales in diagrams as a source of error and bias. • Explain the misuse of grouping in tables and diagrams as a source of error and data manipulation. • Show an awareness of the different features and applications of graphs and tables: o Pie charts reveal the proportions between the different characteristics of the information but do not reveal the population or sample size. o Bar graphs reveal the population or sample size but do not always show the relationship between the categories clearly. o The choice of scale on the axes, and/or the point at which the axes cross, impact on the impression created by the graph. o Tables will often contain more information than graphs, but trends or patterns are less easy to observe. • Answer the questions for which the information was collected.
3.5 Interpret and determine chance (limited to simple probabilities in symmetrical situations)	• Determine the possible outcomes (e.g., using tree diagrams) and calculate the probability of each possible outcome. • Determine actual outcomes and their relative frequencies. • Understand expressions of probability. • Perform simple experiments and count the frequencies of the actual outcomes. • Use the frequencies of the actual outcomes to calculate the relative frequency of each. • Calculate probability based on historical events. • Calculate the probability of non-occurrence, independent and dependent events, mutually exclusive events and multiple independent events.

Element 4: Measurement

Purpose: Make measurements using appropriate measuring tools and techniques and solve problems in various measurement contexts.

Skills:

4.1 Estimate and measure quantities using measuring instruments in various contexts, paying attention to significant figures and margins of error
4.2 Calculate quantities in measurement contexts paying attention to significant figures and margins of error
4.3 Solve measurement problems in various practical and non-practical contexts

Required Standards of Performance:

- Measuring instruments meet the accuracy requirements of the context.
- Readings are accurately recorded within appropriate margins of error and using appropriate units.
- Calculations are performed accurately, keeping units consistent.
- Conversions between units are accurate and appropriate to the context.
- Solutions to problems are validated according to the context, including contextually appropriate rounding and use of units.

Examples of contexts:

- Weigh and measure items and keep records.
- Measure and record medicine doses (use ratio and proportions).
- Read and interpret recipes involving measurements.
- An item, quantity or liquid is provided and measured (a box, for example, must be weighed).
- Situations where a certain quantity must be measured (300g of a quantity, for example, must be weighed out).
- Measure the time it takes a person to run or walk around a field; measure the distance; calculate the speed.

SKILLS	GUIDELINES, SCOPE AND CONTEXTS

4.1 Estimate and measure quantities using measuring instruments in various contexts, paying attention to significant figures and margins of error	• Quantities include:
	  ○ Length, width, height, depth, distance.
	  ○ Time intervals.
	  ○ Temperature.
	  ○ Weight (mass in scientific language).
	  ○ Angles.
	• Measuring instruments include rulers, tape measures, scales, clocks, thermometers, capacity-measuring instruments, protractors, watches and stopwatches.
	• Identify and take account of possible sources of measurement errors.

4.2 Calculate quantities in measurement contexts paying attention to significant figures and margins of error	• Perimeters of polygons and circles. • Areas of triangles, rectangles and circles. • Areas of polygons by dividing them into triangles and rectangles. • Volumes of triangular and rectangular prisms and cylinders. • Surface area of triangular and rectangular prisms and cylinders. • Capacity of rectangular prisms. • Density of shapes.

4.3 Solve measurement problems in various practical and non-practical contexts	• Convert between common units of measure (without the benefit of a conversion table):
	  ○ Between millimetres, centimetres, metres and kilometres.
	  ○ Between millilitres (cubic centimetres) and litres.
	  ○ Between milligrams, grams, kilograms and tonnes.
	  ○ Between seconds, minutes, hours and days.
	• Convert between SI units and Imperial units (with the benefit of a conversion table).
	• Conversion rates (such as SI units to Imperial units).
	• Consumption rates (such as kilometres per litre).
	• Distance, time and speed (such as kilometres per hour).
	• Cost rates (such as Rands per kilogram or Rands per metre).

Element 5: Shape and space

Purpose: Describe and represent objects and the environment in terms of spatial properties and relationships.

Skills:

5.1 Identify and work with geometric figures and solids, including cultural forms and products
5.2 Analyse the properties of geometric figures and solids
5.3 Draw geometric shapes and construct models of solids
5.4 Use the Theorem of Pythagoras to solve problems involving missing lengths in known geometric figures and solids
5.5 Draw different views of simple, regular objects in real-life situations
5.6 Read, interpret and use plans and road maps to show and make sense of real locations, distances and relative positions

Required Standards of Performance

- Two- and three-dimensional shapes are accurately described and compared in terms of their main properties.
- The language of shape and space is used in context.
- Drawings and representations of shapes and solids are consistent with the actual shapes and solids.
- The relative sizes of drawn shapes are consistent with the scale and sizes of the shapes and solids shown.
- Drawings of objects from different viewpoints are consistent with the shape of the object from those views.
- Maps are used effectively to give, and make sense of real locations, distances and relative positions.

SKILLS		GUIDELINES, SCOPE AND CONTEXTS
5.1	Identify and work with geometric figures and solids, including cultural forms and products	• Identify basic geometric shapes. • Identify and work with basic transformations (translations, reflections and rotations). • Identify and work with regular and irregular shapes such as: o Polyhedrons, spheres, cylinders. o Shapes of, and decorations on, cultural products such as drums, pots, mats, buildings, necklaces, architecture and tessellations.
5.2	Analyse the properties of geometric figures and solids	• The properties of triangles, quadrilaterals, regular and irregular polygons, circles and polyhedrons are analysed according to the following, as applicable: o Length, breadth, height, width. o Perimeter. o Diagonal. o Area. o Angle. o Centre, radius, diameter, circumference. o Volume. o Perpendicular. o Parallel. o Lines of symmetry. o Rotational symmetry. o Weight.
5.3	Draw geometric shapes and construct models of solids	• Draw a plan e.g., of your own living space and modify it to suit your needs; draw a floor plan of your learning environment. Show the activity areas.
5.4	Use the Theorem of Pythagoras to solve problems involving missing lengths in known geometric figures and solids	• A ladder leaning against a wall with the legs some distance away from the base of the wall offers an example of a context for working with the Theorem of Pythagoras.
5.5	Draw different views of simple, regular objects in real-life situations	• Left, top and front views of common, regular shapes such as desks, shelves, tables, computer screens, televisions and huts.
5.6	Read, interpret and use plans and road maps to show and make sense of real locations, distances and relative positions	• Make use of map coordinates and grid references to locate places and positions • Use bearings and compass points to find and show direction

Element 6: Patterns and relationships

Purpose: Describe, show, interpret and solve problems involving mathematical patterns, relationships and functions.

Skills:

6.1 Investigate, complete, extend and generate patterns
6.2 Work with and interpret a range of representations of relationships including words, equations, tables of values and graphs
6.3 Represent relationships to solve problems and communicate or illustrate results

Required Standards of Performance:

- Patterns are identified and described in terms of the relationships between the elements of the pattern.
- Generalisations regarding patterns are identified and their validity is verified and explained correctly.
- Patterns are completed and extended in manners consistent with the patterns.
- Relationships between independent and dependent variables are represented clearly and effectively through tables and graphs to facilitate analysis and problem solving.
- Solutions to problems involving patterns and relationships are validated in context.

Examples of functional relationships (independent and dependent variables):

- The time it takes for a journey depends on the distance and the speed travelled.
- The medicine dosage that children receive is determined by their weight or age.
- The cost of electricity is determined by consumption.
- The fee for a bank transaction is determined by the value of the transaction.
- The volume of paint needed to complete a job is determined by the area of the wall to be painted and the spreading rate of the paint.
- The time which a light bulb burns determines the amount of coal that must be burnt to generate the electricity.

SKILLS	GUIDELINES, SCOPE AND CONTEXTS

6.1 Investigate, complete, extend and generate patterns	• Use a range of techniques to determine missing and/or additional terms in a pattern, including the relationship between consecutive terms.
	  ○ Numerical patterns include:
	  – Constant difference patterns (direct proportion and linear relationships), such as the cost of a number of items.
	  – Constant ratio patterns (inverse proportion and exponential relationships), such as growth patterns.
	  ○ Geometric patterns include the tiling patterns used in homes. These are studied to determine how many of the different tiles are needed to complete a pattern.

6.2 Work with and interpret a range of representations of relationships including words, equations, tables of values and graphs	• Recognise, create and describe various patterns and functional relationships.
	• Recognise relationships between input and output variables in terms of their numerical, graphical, verbal and symbolic representations, and convert between the various representations.
	• Identify generalisations of patterns from given expressions i.e., select the correct generalisation from a number of given generalisations
	  ○ Numeric and geometric patterns.
	  ○ Rate of change (the connection between the slope of a line and constant rate of change).
	• Generate patterns using different generalised mathematical forms:
	  ○ Graphs.
	  ○ Formulae.
	• Use expressions and other rules for expressing patterns.
	• Convert from one basic form of representation to another to reveal features of patterns and relationships.
	• Complete a table of values by reading values from a graph e.g., a linear or quadratic function displayed on a graph.
	• Plot a graph from the values in a table of values e.g., a linear or quadratic function.
	• Complete a table of values for a given formulae and/or a description of a relationship.

6.3 Represent relationships to solve problems and communicate or illustrate results	• Choose and develop a representation from tables and graphs.
	• Develop and use simple linear algebraic equations to solve practical problems. These should be limited to situations where linear equations appear in context; they should not be used in the abstract.
	• Select and use simple algebraic formulae to solve problems. They should include changing the subjects of formulae and substituting given values for the unknowns.
	• Solve simultaneous equations in real life problem contexts:
	  ○ e.g., work out whether it is more beneficial to get paid a basic salary plus commission or a fixed salary.
	  ○ e.g., compare the cost of hiring cars from different rental companies with different daily charges and rates per kilometre.
	• Derive information from tables and graphs and answer relevant questions on the data.
	• Use appropriate axes to present information on graphs.
	• Interpret the trends and features of graphs showing mathematical and real-life situations.
	• Develop and solve contextualised problems using various representations such as graphs and tables to understand the purposes and usefulness of each.

SECTION C
FACILITATOR GUIDELINES, ASSESSMENT GUIDELINES AND ACCREDITATION SPECIFICATIONS

1. FACILITATOR GUIDELINES

There are five key principles that should guide developers and facilitators of FML learning programmes:

- FML programmes are learner-centred and discovery-based;
- FML programmes promote communication and participation;
- FML programmes are contextually relevant;
- FML programmes focus on problem solving; and
- FML programmes use tools such as calculators, spreadsheets and standard algorithms to maximise efficiency, but without sacrificing understanding of the underlying mathematical concepts and processes.

1.1 FML programmes are learner-centred and discovery-based

Traditional approaches to teaching mathematics and mathematical literacy have tended to be authoritative and prescriptive. The teacher acted as the mathematical authority and tried to transfer knowledge into passive receptacles by prescribing methods or recipes.

Such learning approaches required learners to interpret or make sense of the teacher's way of understanding a particular concept or of solving a particular problem type, usually within abstract or foreign contexts. This approach encouraged a narrow form of interpretive learning. Learners were expected to understand others' interpretations of knowledge.

The negative effects of such an approach are as follows:

- Learners look to the authority for answers and do not attempt to solve problems themselves. Learners become receptive, passive and reliant.
- The priorities of trying to understand what certain methods actually achieve and why they produce certain results are not emphasised.
- Learners attempt to learn by rote.
- Learners are generally unable to tackle new problems without direction from the authority.
- Learners often build up a fear and dislike for mathematics. This translates into poor performance.

By contrast, progressive ways of developing mathematical literacy rely on learner-centred, discovery-based approaches where realistic problem solving, within accessible contexts, becomes central.

The learner-centred, discovery-based approach acknowledges that learners have prior knowledge and life experiences and that they are not just empty vessels waiting to be filled. The basic principle of this philosophy is that learners construct new knowledge by applying known situations and concepts or by creating new ones.

Learning happens as learners engage in a series of carefully designed learning activities. Problems are solved in a social environment which involves sharing, discussion, comparison and debate. Learners make discoveries for themselves, share experiences with others, engage in helpful debates about methods and solutions, invent new methods, articulate their thoughts and borrow ideas from their peers to solve problems. Using this approach, the facilitator provides learning activities and mediates the learning process.

Learners should be encouraged to look for and develop their own techniques for solving problems. They should also be encouraged to share their understanding of situations. Communication plays a vital role both in making sense of situations and in sharing with others what has been learnt.

Learners should be allowed to develop an understanding of the principles and ideas in each situation before they are introduced to formal terms, symbols and concepts. Formal concepts should only be introduced when learners are able to execute tasks whose nature, meaning and purpose they already know.

This introduction, although brief compared to the extensive amount of literature available, has highlighted the learner-centred approach to designing and offering FML programmes. The approach is presented because it supports the broader aims of the FML Framework vis-à-vis achieving the purposes of FML.

Such an approach:

- promotes self-reliance and self-esteem;
- promotes the confidence to tackle new problems;
- encourages learners to develop problem-solving strategies;
- reduces anxiety and pressure;
- leads to a real understanding of concepts;
- reduces systematic (or silly) errors;
- promotes logical thinking; and
- leads to a more lasting understanding and ability to apply what is learnt.

1.2 FML programmes promote communication and participation

Developers and facilitators of FML learning programmes should recognise the importance of mathematical communication. Mathematical communication is an over-arching process. It includes understanding, expressing and conveying ideas mathematically in order to reflect on and clarify one's thinking, to make convincing arguments, and to reach decisions.

Communication:

- is essential for understanding;
- provides the foundation for learning;
- is the bridge to finding and exchanging ideas, to identifying problems, and to seeking and finding solutions to these problems;
- is essential to working collaboratively; and
- is the link that makes other skills effective.

Mathematical communication is supported through well-designed and carefully monitored group processes.

Learners cooperate to learn in group work. One of the most powerful resources to assist the learning process is group work because learning is essentially a social process. From a very early age, we learn within a social environment, and this resource should be exploited fully in the learning situation.

Facilitators should create a learning environment and learning culture that promotes the verbalisation of ideas, of strategies and of what learners understand. A most powerful reinforcement of their understanding occurs as learners find words to express what is inside their heads. The search for words to articulate thoughts helps to clarify problems and contributes toward greater understanding.

Discussing different ideas and approaches often leads learners to healthy educational debates that transcend the mere mathematics of the problem because it incorporates social, cultural and environmental issues. Discussion of this nature also allows for the transfer of ideas between learners. These could be used as they are or be reworked and reformulated by others.

The verbalisation process also provides the facilitator with a good assessment tool about how well learners are progressing with a particular topic. It allows the facilitator to make decisions about pace: whether to move on, dwell on a topic, or go back.

The verbalisation process assists in:
- formulating ideas;
- clarifying problems;
- reinforcing understanding;
- transferring ideas; and
- assessing progress.

Varied forms of group work should be used. Sometimes learners should work in small groups (2 or 3), sometimes in large groups (5 or 6) and sometimes in one large group. At other times, they could even work individually.

Facilitators should allow learners to use different forms of group work. Sometimes they could work through a problem together from start to finish. At other times, they could work on their own and consult each other from time to time. On other occasions, they could simply compare work at the end. Whatever form is used, facilitators should encourage as much verbalisation as possible.

1.3 FML programmes are contextually relevant

The key to promoting a real and lasting understanding of mathematical concepts is to provide rich and comprehensive learning activities that require problem solving in real-life contexts.

Mathematically literate people make sense of situations, solve problems, and offer and validate solutions within specific contexts. In real-world applications, people need to be able to work within a range of different contexts and transfer the skills they developed in familiar contexts to unfamiliar contexts. However, studies show that these skills are not easily transferred. We cannot assume that learners will automatically transfer skills learnt in familiar contexts to unfamiliar contexts.

Thus, one goal in a mathematical literacy programme should be to expose learners to a wide range of contexts in order to develop a solid foundation in the mathematics involved and to improve transfer skills. However, when learners find themselves in unfamiliar contexts, whether in the world of work or in occupational learning programmes, they will generally need some form of coaching to apply mathematical learning to those particular contexts.

For this reason, although FML programmes should expose learners to a variety of contexts, such programmes will not necessarily enable learners to apply their skills in all contexts. However, it is reasonable to expect, when learners are confronted by unfamiliar contexts – either in occupational learning programmes or at work – that they will receive the assistance they need to apply their knowledge to the new contexts.

Note: The FML Framework does not aim to cover the full range of contexts that people will meet in life, work and learning. Furthermore, the framework is based on the assumption that particular occupational contexts will be addressed within those particular occupations.

Given the nature of mathematical literacy and the principles of adult learning, it is recommended that learning programmes are based on activities that are:

- realistic and relevant to the context of learners; and
- vary in complexity.

Relevant, realistic and context-led learning activities

Learning activities should involve contexts that are relevant to the learners. There should be clear connections to other disciplines, to real life, to the workplace and to the various areas of mathematics. Learners need to make social, historical or personal links and connect with other learning areas. Mathematics connects to virtually everything, and good learning activities make these connections explicit.

When the learning activities relate to the workplace or real-life situations, then mathematical literacy becomes more meaningful and learners will be able to make sense of, and remember, the concepts involved. Learners need to be able to make links between what they already know and new learning.

The emphasis in mathematical literacy is on working within a variety of contexts. Thus, learning programmes should enable learners to work within real-life contexts, while at the same time ensuring the development of those mathematical concepts that are fundamental to mathematical literacy. Furthermore, learners learn more effectively when they encounter problems in contexts that invite full use of their personal strengths and do not rely simply on a narrow range of skills.

Varying complexity

Different kinds of problems and different contexts demand more complex problem-solving skills and learners should be equipped to deal with these more complex levels.

Some of the factors that determine the complexity of the problems include:

- how familiar the contexts are;
- the instruments and language used;
- related concepts;
- the number of steps involved;
- how organised the data is;
- the extent to which the information is complete and accurate;
- the degree of certainty or uncertainty;
- the number of concurrent factors; and
- how routine the problem solving process is.

Problem solving becomes progressively more complex when we consider the mathematical concepts themselves. For example, when we think about the concept of measurement, we can see that it gets more complicated as we move:

- from measuring length (such as distance and height), either directly or by observing similarity;
- to measuring area (of regular and irregular shapes), by using squares, cutting and rearranging surface area and using scales;
- to measuring volume and capacity;
- to working out dimensions;
- to changing the shapes of containers while keeping volumes the same (such as by changing cylinders to prisms); and finally
- to measuring pressure in relation to volume (such as in car tyres).

When we consider problem types we may require learners to find missing dimensions if only some are given. We might also ask learners to calculate how many smaller containers fit into a bigger one. In the latter example, we need to consider the levels of accuracy we require in the measurement, what percentage waste is acceptable, whether a better fit would be possible if the geometric shape were changed, and so on. We might also consider other measurements such as 'business confidence', inflation or growth in property values. In all instances, we would create an index to measure against.

In real life, people have to cope with problem-solving situations of varying levels of complexity. Sometimes the problems can be solved in one or two easy steps. On other occasions people require several steps. Sometimes all the relevant information is readily at hand. At other times problem-solvers have to gather information, identify missing information and evaluate its relevance and usefulness. This is a reality about real-life problem solving, and thus, learning activities in FML programmes should include problems of varying levels of complexity.

1.4 FML programmes focus on problem solving

Problem-solving activities combine the interest in, and usefulness of, mathematics in applications that build on learners' own experiences.

There are two main considerations in problem solving:

- Learners learn *through* problem solving. Studies show that learners are able to develop a real and lasting understanding of mathematics when they learn through an approach that has problem solving as its central feature. Learners are better able to understand the various concepts when they encounter them in problems they can identify with. They are also better able to make the necessary links between related concepts. Thus, problem solving ought to be a major part of any learning methodology for mathematical literacy.
- Learners need to learn *about* problem solving. Problem solving is important as a methodology. It is also a critical skill in its own right. While problem solving is not the only skill to develop in mathematical literacy, it is a critical skill. Furthermore, problem solving becomes a central component of mathematical literacy, which is defined as the application of mathematics to real-life contexts.

Problem solving, therefore, needs to be a central feature of Foundational Mathematical Literacy because of its value as a methodology to aid learning and because of its importance as a skill. Learners should be confronted by a large variety of problem types and they should be allowed to solve the problems using their own ideas and methods.

The steps in the problem solving process have been frequently described (cf. Polya, 1945/1980) as:

- defining the goals;
- analysing the situation and constructing a mental picture of it;
- devising a strategy and planning the steps to take;
- executing the plan, including its control (and, if necessary, modifying the strategy); and
- evaluating the result.

The table below illustrates how these steps have been expanded:

Defining the goals	• Set goals. • Decide which goals must be reached and specify the reasons for the decision. • Recognise which goals/wishes are contradictory and which are compatible. • Assign priorities to goals/wishes.
Analysing the situation	• Select, obtain and evaluate information. Decide: o what information is required, what is already available, what is still missing, and what is not needed; o where and how you can obtain the information; and o how you should interpret the information. • Identify the people (and with what knowledge and skills) who should be involved in solving the problem. • Select the tools to be used. • Recognise conditions (such as time restrictions) that need to be taken into account.

Planning the solution	• Decide on the steps to be taken. • Decide on the sequence of steps. • Coordinate work and deadlines. • Make a comparative analysis of alternative plans and decide which plan is most suitable for achieving the goals. • Adapt the plan to changed conditions if necessary. • Decide on a plan
Executing the plan	• Carry out the individual steps.
Evaluating the results	• Assess whether, and to what extent, the target has been reached. • Recognise the mistakes made. • Identify the reasons for the mistakes. • Assess the consequences of the mistakes

(Reeff, J-P., Zabal, A. & Blech, C. 2006. *The Assessment of Problem-Solving Competencies A draft version of a general framework*. Deutsches Institut für Erwachsenenbildung).

1.5 FML programmes use tools such as calculators, spreadsheets and standard algorithms to maximise efficiency, but without sacrificing understanding of the underlying mathematical concepts and processes

The experience of mathematics, for many learners, is nothing more than learning and applying standard algorithms that work efficiently.

Standard algorithms, calculators and spreadsheets are very useful for accurate computations. However, they can be counter-productive from an educational perspective. Educational objectives ought to prioritise thinking skills and understanding, and the blind use of standard algorithms can lead to the opposite.

Although the use of standard algorithms, calculators and spreadsheets has an important place in mathematical literacy, facilitators should take care to ensure that learning activities promote thinking and understanding. Therefore, facilitators should control the use of these tools.

It is important for learners to develop computational strategies that they understand. This enables them to make sense of what they are doing. Devising their own computational strategies promotes mathematical thinking. Thus, there will be times when facilitators should avoid the use of tools such as standard algorithms, calculators and spreadsheets to challenge the learners to devise (and share with others) their computational strategies. This should form a significant part of mathematical literacy sessions. The logic and concepts that underpin the algorithms then become clear.

There may be times when facilitators choose to reveal or teach standard algorithms, but facilitators should decide when it would be useful and appropriate. Often, learners' own strategies are just as efficient as standard algorithms. In other words, it is not an objective of mathematical literacy to teach standard algorithms, but they are useful tools for quick computations and so can be incorporated when needed. The important thing is to reveal these tools and strategies to learners only after they have gained an understanding of the key concepts and processes and can appreciate the need for methods that are more efficient.

On the other hand, there will be times when the learning activity will be compromised by the absence of quick, efficient tools such as standard algorithms, calculators or spreadsheets. For example, when learners are engaged in complex problem-solving exercises, we want to promote the higher-order skills of thinking, reasoning and decision making. We do not want learners to become distracted by the intricacies of difficult calculations at those times. We should not pose problems merely to give practice in computation. Rather, we should set problems that will enable learners to develop logical problem-solving strategies and to provide meaningful contexts through which to learn new concepts. Efficient tools become important then because they allow learners to focus on higher-order problem-solving skills.

2. ASSESSMENT GUIDELINES

INTERNAL CONTINUOUS ASSESSMENT

Facilitators of FML programmes should engage in continuous assessment of learner progress to ensure that gaps are discovered early and suitable learning activities are used to address such gaps. Assessments should be designed to address each of the six elements comprehensively, as well as to ensure meaningful integration across the elements.
The following tables provide facilitators with a useful reference point for what learners should be achieving in relation to each element.

Number	Use numbers in a variety of forms to describe and make sense of situations, and to solve problems in a range of familiar and unfamiliar contexts.
What learners must be able to do:	Criteria against which performance must be measured:
• Use numbers to describe and make sense of real-life situations • Read, interpret and use different numbering conventions in different contexts and identify the ways in which different conventions work • Interpret and use numbers written in exponential form including squares and cubes of natural numbers and the square and cube roots of perfect squares and cubes • Do calculations in various situations using a variety of techniques • Solve problems involving ratio and proportion • Solve problems involving fractions, decimals and percentages	• Problem-solving strategies are based on a correct interpretation of the context. • Calculations are performed accurately and according to the conventions governing the order of operations. • Methods are presented in a clear, logical and structured manner, using mathematical symbols and notation consistent with mathematical conventions. • Methods used are efficient, logical, internally consistent and justified by the context. • The degree of accuracy of solutions is justified by the context. • Solutions are evaluated and validated in terms of the context, and numbers are rounded appropriately to the problem situation.
NOTE: Refer to the FML Framework for the full description of scope and contexts to be covered.	

Finance	Manage personal finances using financial documents and related formulae.
What learners must be able to do:	Criteria against which performance must be measured:
• Read and interpret financial information presented in a range of documents in personal and familiar contexts • Identify, classify and record sources of income and expenditure • Plan and monitor personal finances • Evaluate options when purchasing products and services • Determine the impact of interest, depreciation, inflation, deflation and taxation on personal finances	• Interpretations of personal financial documents are consistent with recorded facts. • Financial information is recorded and organised clearly, accurately and according to general finance-recording techniques and principles. • Personal budgets reflect the financial situation in sufficient detail for planning and monitoring personal finances. • Costs of products and services are evaluated using a variety of issues. These include affordability, personal needs and accuracy of advertising claims. • Personal finances are monitored in terms of various influences, including income, expenditure, investments, loans, taxation and inflation.
NOTE: Refer to the FML Framework for the full description of scope and contexts to be covered.	

Data & Chance	Collect, display and interpret data in various ways and solve related problems.
What learners must be able to do:	Criteria against which performance must be measured:
• Collect data from various sources and in various ways • Classify, organise and summarise data • Display data using tables, graphs and charts • Analyse and interpret data to draw conclusions and make predictions • Determine and interpret simple chance in everyday contexts	• Data collection techniques are appropriate to the context, type of data and purpose. • Data is classified, organised and summarised appropriately so that it promotes meaningful analysis. • Data is displayed using techniques appropriate to the type of information and context. • Data displays are consistent with collected data, promote ease of interpretation and minimise bias. • Interpretations and predictions are verified by the data and observed trends. They take into account possible sources of error and data manipulation. • Simple probabilities are determined accurately and statements of chance are correctly interpreted in context.
NOTE: Refer to the FML Framework for the full description of scope and contexts to be covered.	

Measurement	Make measurements using appropriate measuring tools and techniques and solve problems in various measurement contexts.
What learners must be able to do:	Criteria against which performance must be measured:
• Estimate and measure quantities using measuring instruments in various contexts, paying attention to significant figures and margins of error • Calculate quantities in measurement contexts paying attention to significant figures and margins of error • Solve measurement problems in various practical and non-practical contexts	• Measuring instruments used meet the accuracy requirements of the context. • Readings are accurately recorded within appropriate margins of error and using appropriate units. • Calculations are performed accurately, keeping units consistent. • Conversions between units are accurate and appropriate to the context. • Solutions to problems are validated according to the context, including contextually appropriate rounding and use of units.
NOTE: Refer to the FML Framework for the full description of scope and contexts to be covered.	

Shape & Space	Describe and represent objects and the environment in terms of spatial properties and relationships.
What learners must be able to do:	Criteria against which performance must be measured:
• Identify and work with geometric figures and solids, including cultural forms and products • Analyse the properties of geometric figures and solids • Draw geometric shapes and construct models of solids • Use the Theorem of Pythagoras to solve problems involving missing lengths in known geometric figures and solids • Draw different views of simple, regular objects in real-life situations • Read, interpret and use plans and road maps to show and make sense of real locations, distances and relative positions	• Two- and three-dimensional shapes are accurately described and compared in terms of their main properties. • The language of shape and space is used in context. • Drawings and representations of shapes and solids are consistent with the actual shapes and solids. • The relative sizes of drawn shapes are consistent with the scale and sizes of the shapes and solids shown. • Drawings of objects from different viewpoints are consistent with the shape of the object from those views. • Maps are used effectively to give and make sense of real locations, distances and relative positions.
NOTE: Refer to the FML Framework for the full description of scope and contexts to be covered.	

Patterns & Relationships	Describe, show, interpret and solve problems involving mathematical patterns, relationships and functions.
What learners must be able to do:	Criteria against which performance must be measured:
• Investigate, complete, extend and generate simple number and geometric patterns • Work with and interpret a range of representations of relationships including words, equations, tables of values and graphs • Represent relationships to solve problems and communicate or illustrate results	• Patterns are identified and described in terms of the relationships between the elements of the pattern. • Patterns are expressed in general terms where possible. • Patterns are completed and/or extended in keeping with the general patterns. • Relationships between independent and dependent variables are represented clearly and effectively through tables and graphs to facilitate analysis and problem solving. • Solutions to problems involving patterns and relationships are validated in context.
NOTE: Refer to the FML Framework for the full description of scope and contexts to be covered.	

EXTERNAL SUMMATIVE ASSESSMENT

General Information

The Foundational Learning Competence Assessment is a national assessment offered in the two learning areas of Foundational Communication and Foundational Mathematical Literacy. Its purpose is to provide a quick and efficient means to benchmark the broad competence level of an individual in the two Foundational Learning areas, in support of successful occupational training.

This national assessment has the following features:

- It is a machine scored, item-based, multiple choice format assessment.
- It is available at regular intervals, with quick delivery of results.
- It is administered by an external Assessment Quality Partner appointed by the QCTO.
- Successful candidates are awarded a statement of results by the QCTO.

Success in the Foundational Learning assessment in both learning areas is compulsory for final award of any occupational qualifications at NQF Levels 3 and 4. Curriculum designers for occupations at NQF level 2 will have the option to decide whether FML is a requirement for award or not.

Candidates can enter the assessments before or during their occupational training. If successful in a learning area, they do not need to undertake a Foundational Learning programme in that area. If unsuccessful, they undertake the relevant learning programme and then re-take the relevant Foundational Learning Assessment.

The Foundational Learning Competence Assessment for Foundational Mathematical Literacy

The purpose of this assessment is to determine whether a learner has sufficient competence and skills in mathematical literacy to engage successfully with formal occupational training up to NQF Level 4.

The assessment is based on the Foundational Mathematical Literacy Curriculum according to a construct developed by the assigned Assessment Quality Partner in such a way as to ensure the sample of questions fairly represents all six elements and contains a spread of questions at varying levels of complexity.

Providers should advise learners who are below ABET Level 3 in English language competence that they are unlikely to be able to deal with the literacy demands of the test.

Learners in programmes should be prepared for the assessment through:

- Familiarisation with the format and instructions of the paper.
- Practise in reading and understanding multiple choice questions, in terms of what these are assessing and how to 'think through' the options given.
- Practise in using time efficiently in order to complete the paper.